**Problem Set 4**

**Note: You can express your probabilities either in terms of decimals or percentages. Thus, having a probability of 0.2 is the same thing as having a probability of 20%. In my answers below, I have chosen to use the decimal format. Since they are both equivalent, you can choose to express your answers in whichever form you prefer.**

**Problem 1.** The time taken to assemble a car in a certain plant is a random variable having a normal distribution of 20 hours and a standard deviation of 2 hours. What is the probability that a car can be assembled at this plant in a period of time   
  
a) less than 19.5 hours?

**Let T denote the time taken to assemble a car. Since T is normal and hence continuous, P{T < 19.5} = P{T <= 19.5} = normdist(19.5, 20, 2, 1} = 0.40 (approximately).**

b) between 20 and 22 hours?

**We seek P{20 <= T <= 22} = P{T <= 22} - P{T <= 20} = normdist(22, 20, 2, 1) - normdist(20, 20, 2, 1) = 0.84 – 0.5 = 0.34 (approximately).**

**Problem 2.** X is a normally distributed variable with mean μ = 30 and standard deviation σ = 4. Find   
  
a) P(x < 40)

**= normdist(40, 30, 4, 1) = 0.994 (approximately).**

b) P(x > 21)

**= 1 – P{X <= 21} = 1 – normdist(21, 30, 4, 1) = 1 – 0.012 = 0.988 (approximately).**

c) P(30 < x < 35)

**= P{X <= 35} - P{X <= 30} = normdist(35, 30, 4, 1) – normdist(30, 30, 4, 1) = 0.894 – 0.5 = 0.394 (approximately)**

**Problem 3.** A radar unit is used to measure speeds of cars on a motorway. The speeds are normally distributed with a mean of 90 km/hr and a standard deviation of 10 km/hr. What is the probability that a car picked at random is travelling at more than 100 km/hr?

**Let V = the velocity of the car. We seek P{V >= 100} = 1 – P{V <= 100} = 1 – normdist(100, 90, 10, 1) = 1 – 0.841 = 0.159 (approximately).**

**Problem 4.** When you check into the hospital for a routine procedure, a nurse takes your medical history. Suppose the time required to take the medical history is a normally distributed random variable with a mean of 12 minutes and a standard deviation of 2.5 minutes. What is the probability that the nurse will finish taking your history in 9 minutes or less?

**Let T denote the time required. We seek P{T <= 9} = normdist(9, 12, 2.5, 1) = 0.115 (approximately).**

**Problem 5**. Entry to a certain University is determined by a national test. The scores on this test are normally distributed with a mean of 500 and a standard deviation of 100. Tom wants to be admitted to this university and he knows that he must score better than at least 70% of the students who took the test. Tom takes the test and scores 585. Will he be admitted to this university?

**We want to know if Tom’s score is above the 70th percentile. In other words, we want to know the 70th Percentile Score which is the lowest score that 70% of the students do NOT exceed. For this calculation, we use the norminv function:**

**70th Percentile Score = Norminv(0.7, 500, 100) = 552.44 or, rounding up, 553. Since Tom’s score exceeds 553, he is above the 70th percentile and hence will be admitted.**

**Problem 6.** The length of similar components produced by a company are approximated by a normal distribution model with a mean of 5 cm and a standard deviation of 0.02 cm. If a component is chosen at random

**Let L denote the length of the randomly chosen part.**  
a) what is the probability that the length of this component is between 4.98 and 5.02 cm?

**P{4.98 <= L <= 5.02} = P{L <= 5.02} - P{L <= 4.98} =**

**normdist(5.02, 5, 0.02, 1) - normdist(4.98, 5, 0.02, 1) = 0.84 – 0.16 = 0.68 (approx)**

b) what is the probability that the length of this component is between 4.96 and 5.04 cm?

**P{4.96 <= L <= 5.04} = P{L <= 5.04} - P{L <= 4.96} =**

**normdist(5.04, 5, 0.02, 1) - normdist(4.96, 5, 0.02, 1) = 0.978 – 0.023 = 0.955 (approx)**

**Problem 7.** A piston in a heart valve must fit into a sleeve. Due to inevitable variations in manufacturing processes, the diameters of pistons and sleeves vary somewhat. The diameter of a sleeve, denoted D, is normally distributed with a mean of 0.0650 inches and a standard deviation of 0.0002 inches. The diameter of a piston, denoted d, is normally distributed with a mean of 0.0600 inches and a standard deviation of 0.0002 inches.

A critical factor that determines how long the heart valve will last is the clearance between the piston and the sleeve, defined as

Clearance = C = D - d.

1. Is the clearance, C, normally distributed and if so, what are its mean and standard deviation?

**The clearance, C, is a weighted sum of normally distributed random variables---the weight on D is +1 and the weight on d is -1. Hence C is normally distributed with**

**a mean of : (+1) x 0.065 + (-1) x 0.060 = 0.005 inches**

**a variance of: (+1)2 x (0.0002)2 + (-1)2 x (0.0002)2 = 0.00000008 = 8 X 10 -8**

**a standard deviation of sqrt(8 X 10 -8 ) = 0.0002828 (approximately)**

1. The nominal design clearance is 0.005 inches, but the heart valve will function acceptably so long as the actual clearance is held to within plus or minus 0.0002 inches of the nominal design standard. What fraction of the piston/sleeve assemblies will meet the required tolerance?

We seek the probability that a randomly chosen piston/sleeve assembly will meet the required tolerance. In other words,

**P{0.0048 <= C <= 0.0052} =**

**normdist(0.0052, 0.005, 0.0002828, 1) - normdist(0.0048, 0.005, 0.0002828, 1) =**

**0.76 – 0.24 = 0.52. So only 52% of the assemblies meet the required standards.**

**Problem 8.** Skateboards are assembled in a particular manufacturing plant on an assembly line by three workers. Worker 1 performs the first three assembly steps, worker 2 performs the next three, and worker 3 performs the final three.   
  
a) Suppose the time for worker 1 to perform each of his three assembly steps is a normally distributed random variable. The first step averages 30 seconds, the second step averages 20 seconds, and the third step 10 seconds. The standard deviations for each of the three steps are 6 seconds, 4 seconds, and 2 seconds respectively. What is the mean and standard deviation of the total time it takes for worker 1 to perform his three assembly steps?

**Let T = total time and let A, B, and C denote the times of his first, second, and third step respectively.**

**T = A + B + C, so T is the weighted sum of three normally distributed random variables where the weights are all +1. T is, therefore, normally distributed with**

**Mean = 30 + 20 + 10 = 60**

**Variance = 62  + 42 + 22 = 36 + 16 + 4 = 56**

**Standard deviation = sqrt(56) = 7.48 (approximately)**

b) The total time for worker 2 to perform her three assembly steps is normally distributed with an average of 70 seconds and a standard deviation of 4 seconds. Suppose worker 1 and worker 2 each start their part in the assembly operation (on different and successive skateboards) at the same time. What is the probability that worker 2 finishes first and is left waiting (assuming no work in process inventory between the two stations) with nothing to do until worker 1 finishes?

**Let T denote the time of the first worker and S denote the time of the second worker. Worker 2 finishes first if and only if S < T which is equivalent to S – T < 0.**

**Let W = S - T. W is a weighted sum of random variables where the weight on S is +1 and the weight on T is -1. W is, therefore, normally distributed with**

**Mean = 70 - 60 = 10**

**Variance = 16 + 56 = 72**

**Standard deviation = sqrt(72) = 8.49 (approximately)**

**We seek P{W <= 0} = normdist(0, 10, 8.49, 1) = 0.12 (approximately)**

c) Given the situation as stated in part b, what is the probability that worker 1 finishes first and is blocked (he can’t start work on a new skateboard until he can hand the one he just finished off to worker 2)?

**Worker 1 finishes first if and only if worker two’s time, S, is larger than worker one’s time, T. In other words, if and only if S – T > 0. Thus, we seek P{W > 0}.**

**P{W > 0} = 1 - P{W <= 0} = 1 - 0.12 = 0.88.**

**Problem 9.** Suppose that the weight of an individual penny is normally distributed with a mean and standard deviation of 2.5g and 0.06g respectively.

1. A roll of pennies contains 50 pennies. What is the probability that the weight of the roll of pennies (ignoring the weight of the wrapper) is between 124 and 126 grams?

**Let X1 through X50 denote the weights of the 50 pennies respectively.**

**Let W = X1 + X2 + ….. + X50**

**Then W is a weighted sum of normally distributed random variables (all the weights are +1) and hence is normally distributed itself with**

**Mean = 50 x 2.5 grams = 125 grams**

**Variance = 50 x 0.062 = 0.18**

**Standard Deviation = sqrt(0.18) = 0.424 (approximately)**

**P{124 <= W <= 126} = normdist(126, 125, 0.424, 1) - normdist(124, 125, 0.424, 1) = 0.98 (approximately)**

1. What is the probability that the average weight of the 50 pennies is between 2.49 and 2.51 grams? Hint: The average is a normally distributed random variable---what are its mean and standard deviation (variance)?

**The weights are all now 1/50, so**

**Mean = 2.5**

**Variance = (1/50)2 x 0.062 + (1/50)2 x 0.062 + … + (1/50)2 x 0.062**

**= 50 x (1/50)2 x 0.062 = (1/50) x 0.062 = 0.000072**

**Standard deviation = sqrt(0.000072) = 0.0085 (approximately)**

**P{2.45 <= W <= 2.55} =**

**normdist(2.51, 2.5, 0.0085,1) - normdist(2.49, 2.5, 0.0085, 1)**

**= 0.76 (approximately)**

1. Repeat part b, but now instead of 50 pennies, suppose you are considering the average weight of only 5 pennies.

The only difference in the mean/variance calculations from part b is in the variance. Now, since we are taking the average of 5 pennies:

**Variance = 5 x (1/5)2 x 0.062 = (1/5) x 0.062 = 0.00072**

**Standard deviation = sqrt(0.00072) = 0.027 (approximately).**

**P{2.45 <= W <= 2.55} =**

**normdist(2.51, 2.5, 0.027,1) - normdist(2.49, 2.5, 0.027, 1)**

**= 0.29 (approximately)**

1. Repeat part b, but now instead of 50 pennies, suppose you are considering the average weight of 500 pennies.

**The only difference in the mean/variance calculations from part b is in the variance. Now, since we are taking the average of 500 pennies:**

**Variance = 500 x (1/500)2 x 0.062 = (1/500) x 0.062 = 0.0000072**

**Standard deviation = sqrt(0.0000072) = 0.0027 (approximately).**

**P{2.45 <= W <= 2.55} =**

**normdist(2.51, 2.5, 0.0027,1) - normdist(2.49, 2.5, 0.0027, 1)**

**= 0.9998**

1. Can you draw any conclusions from your calculations in parts b thru d?

**Yes, as the number we take the average of gets larger, we get a more and more precise estimate of the true mean.**